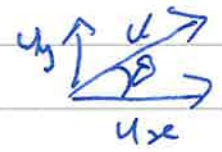
velocity Transformation

S' moves velocity v along the x-axis



$$u_x' = \frac{u_x - v}{1 - \frac{v u_x}{c^2}}$$

and $$u_y' = \sqrt{1 - v^2/c^2} \cdot \frac{u_y}{1 - \frac{v u_x}{c^2}}$$

$u_x = u \cos\theta$

$u_y = u \sin\theta$

then $$u_x' = \frac{u\cos\theta - v}{1 - \frac{uv}{c^2}\cos\theta}$$

$$u_y' = \sqrt{1 - v^2/c^2} \cdot \frac{u \sin\theta}{1 - \frac{uv}{c^2}\cos\theta}$$

(1) Problems $u < c$, $v < c$

for $\frac{uv}{c^2} < 1$

Relativistic boost along the x axis with vel. v gives

$$u_x' = \frac{u\cos\theta + v}{1 + \frac{uv}{c^2}\cos\theta}$$

$$u_y' = \sqrt{1 - v^2/c^2} \cdot \frac{u \sin\theta}{1 + \frac{uv}{c^2}\cos\theta}$$

(2) $u > c$

Same formula applies with boosts with $u < c$

check for homogeneity

$$u_x' = \frac{u_x - v}{1 - \frac{v u_x}{c^2}}$$

$$\therefore \quad u_x - v = u_x' - \frac{v}{c^2} u_x' u_x$$

$$\therefore \quad u_x \left(1 + \frac{v u_x'}{c^2}\right) = u_x' + v$$

$$\& \quad u_x = \frac{u_x' + v}{1 + \frac{v u_x'}{c^2}} \quad \checkmark$$

$$u_y = \frac{1}{\sqrt{1 - v^2/c^2}} \cdot u_y' \left(1 - \frac{v}{c^2} \frac{(u_x' + v)}{1 + \frac{v u_x'}{c^2}}\right)$$

$$= \frac{u_y'}{\sqrt{1 - v^2/c^2}} \left(1 - \left\{\frac{v u_x'}{c^2} + \frac{v^2}{c^2}\right\} \Big/ \left(1 + \frac{v u_x'}{c^2}\right)\right)$$

$$= \frac{u_y'}{1 + \frac{v u_x'}{c^2}} \left(1 + \frac{v u_x'}{c^2} - \frac{v u_x'}{c^2} - v^2/c^2\right) \sqrt{1 - v^2/c^2}$$

$$= \sqrt{1 - v^2/c^2} \; \frac{u_y'}{1 + \frac{v u_x'}{c^2}}$$

which checks

Now for Bradfords

$\theta = 0$	$u_x = u$
$\theta = \pi/2$	$u_x = 0$
$\theta = \pi$	$u_x = -u$

$\theta = 0$ $\quad u_x' = \dfrac{u + v}{1 + uv/c^2}$ $\quad = u$ for $v = 0$

$u_y' = 0$ $\quad = c$ for $v = c$

$$= \frac{u + c/2}{1 + \frac{1}{2} u/c} = \frac{2u + c}{2c + u} \cdot c \quad \text{for } v = c/2$$

$\theta = \pi/2$. $\quad u_x' = \dfrac{v}{\gamma}$ $\Big\}$ $= 0$ for $v=0$, $= c$ for $v=c$

$u_y' = \sqrt{1-v^2/c^2}\,.$ $\Big\}$ u for $v=0$, 0 for $v=c$

$\tan\theta' = \dfrac{u\sqrt{1-v^2/c^2}}{v}$

$\theta = \pi$ $\quad u_x' = \dfrac{-u+v}{1-\frac{uv}{c^2}} = -u$ for $v=0$

$= c$ for $v=c$.

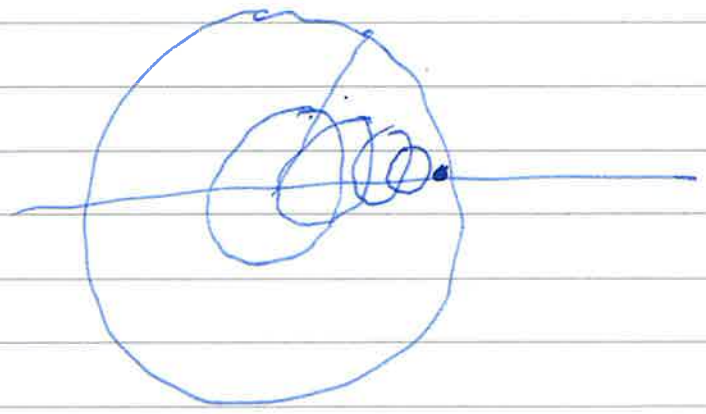
$u_y' = 0$.

~~θ for u this~~.

$u_x' = 0$ when $u\cos\theta + v = 0$

for $\cos\theta = -v/u$

θ for u this



Most value for u_y'

$$\frac{du_y'}{d\theta} = \frac{u}{\gamma}\cdot\left[\left(1+\frac{uv}{c^2}\cos\theta\right)\cos\theta + \sin^2\theta\cdot\frac{uv}{c^2}\right] \Big/$$

$= 0$ when $\cos\theta + \frac{uv}{c^2}\cos^2\theta + \frac{uv}{c^2}\sin^2\theta = 0$

or $\cos\theta + \frac{uv}{c^2} = 0$ $\quad$ or $\cos\theta = -\frac{uv}{c^2} < 1$

& $\sin\theta = \sqrt{1-u^2v^2/c^4}$

$$\therefore u_y'(\max) = \frac{\sqrt{1-v^2/c^2}\cdot u\sqrt{1-u^2v^2/c^4}}{1-\frac{u^2v^2}{c^4}}$$

and $2u \Rightarrow \dfrac{2\sqrt{1-v^2/c^2}\cdot u\sqrt{1-u^2v^2/c^4}}{1-u^2v^2/c^4} = \dfrac{2u\sqrt{1-v^2/c^2}}{\sqrt{1-u^2v^2/c^4}}$

But for 2a we have

$$2a = \frac{u+v}{1+uv/c^2} - \frac{v-u}{1-uv/c^2}$$

$$= \frac{(u+v)(1-uv/c^2) - (v-u)(1+uv/c^2)}{1-u^2v^2/c^4}$$

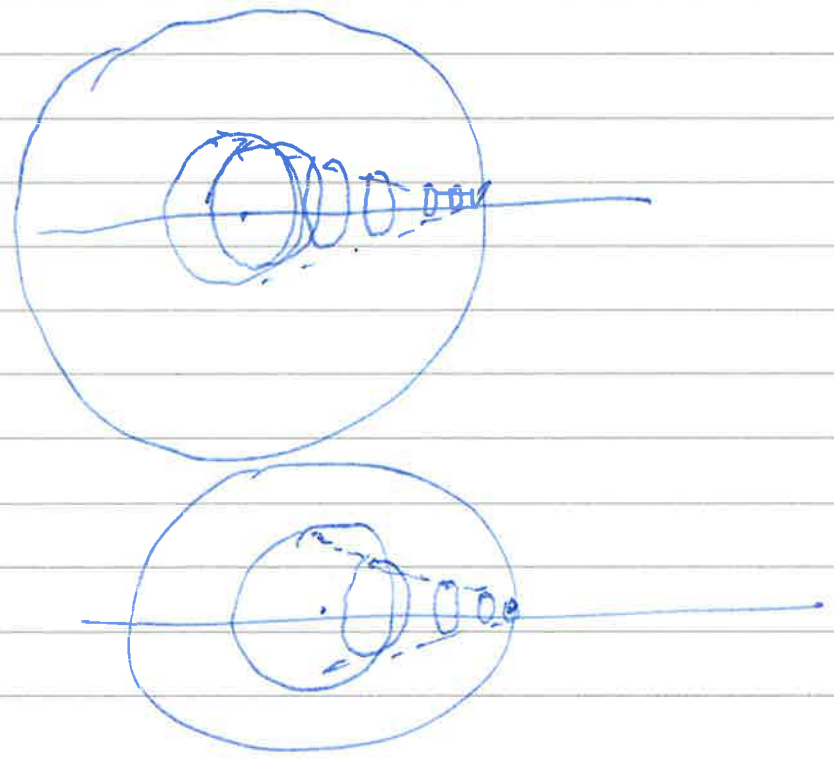
$$= \frac{u - u^2v/c^2 - v - uv^2/c^2 + u + u^2v/c^2}{1-u^2v^2/c^4}$$

$$= \frac{2u(1-v^2/c^2)}{1-u^2v^2/c^4}$$

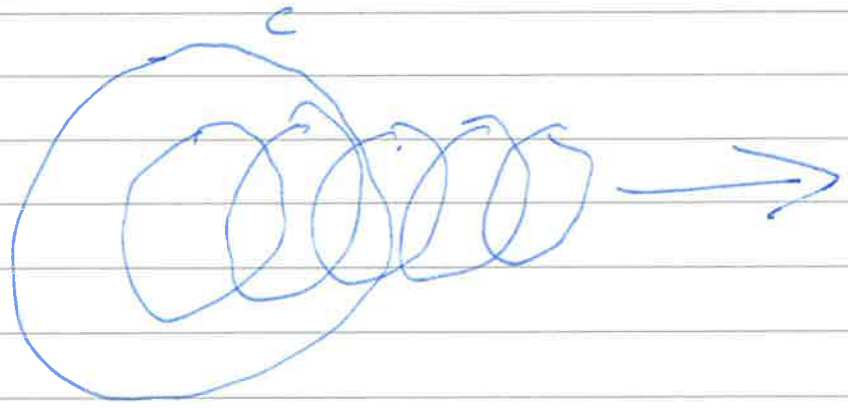
$$\therefore \frac{2b}{2a} = \frac{\sqrt{1-v^2/c^2}\ \sqrt{1-u^2v^2/c^4}}{1-v^2/c^2}$$

$$= \sqrt{\frac{1-u^2v^2/c^4}{1-v^2/c^2}} \to \infty \text{ as } v \to c.$$

so factor is the ratio.



Spherical Bodies



Now take a Torpedo case

$\theta = 0$, $u_n' = u$ for $v = 0$

$= c$ for $v = c$.

$\theta = \pi$ $u_n' = -u$ for $v = 0$

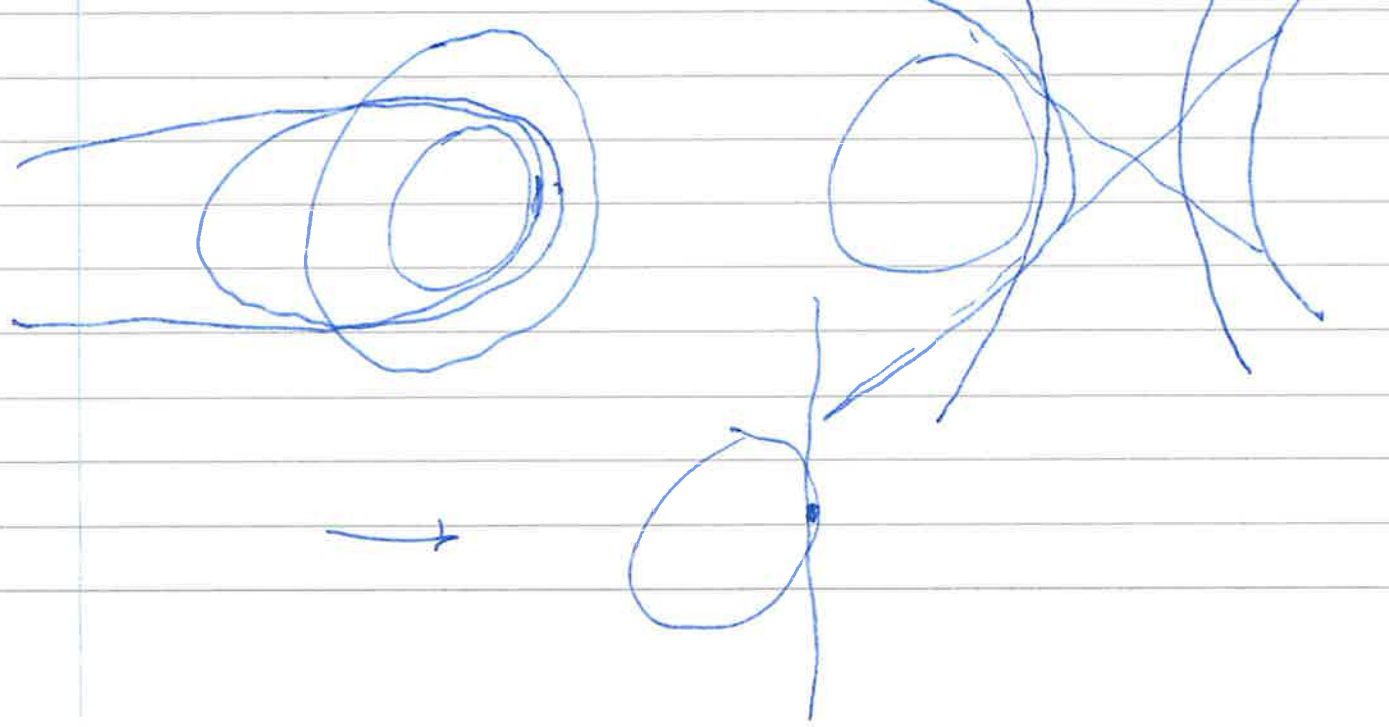
$= c$ for $v = c$.

Transcends for $\theta = \pi$ occurs at $v = \frac{c}{u} \cdot c$. $< c$

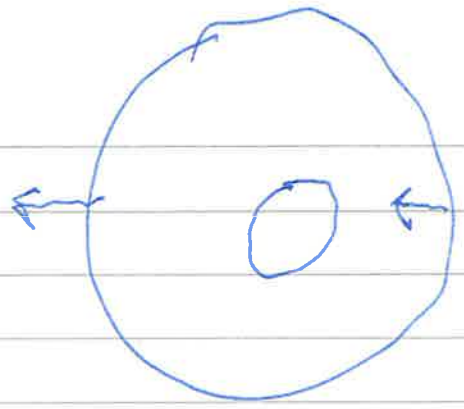
Transcends at angle θ occurs for $v \cos\theta = -\frac{c}{u}$

i.e. $v = -\frac{c}{u} \cdot c \cdot \sec\theta$.

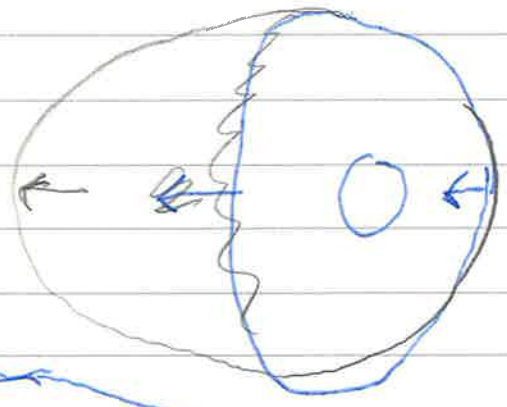
So pressure is less than '.



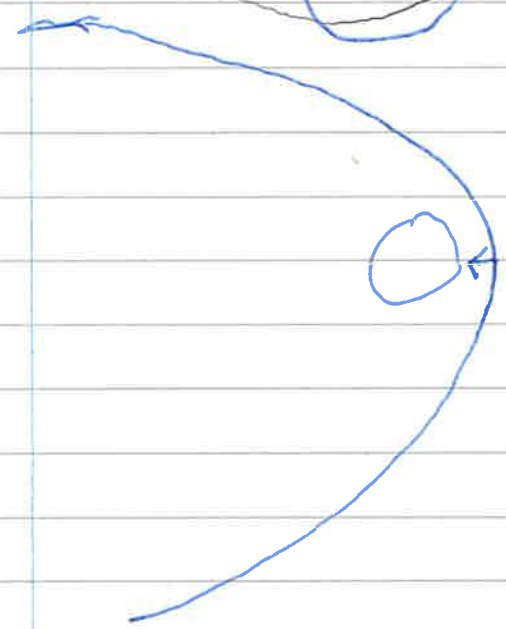
(1)



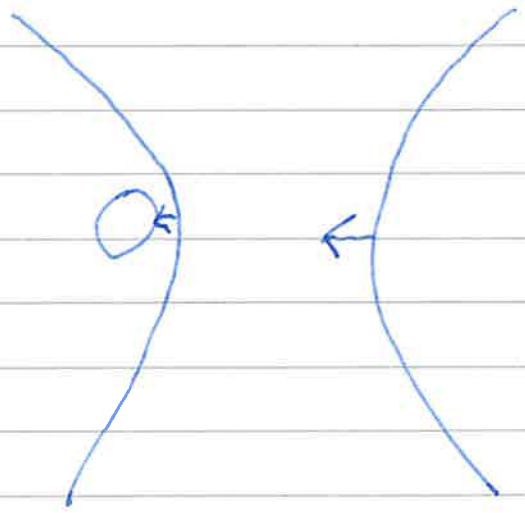
(2)



(3)



(4)



(5)

At transcendence.

$$\cos\theta = -\frac{c}{u}\cdot\frac{c}{v} = -\frac{c^2}{uv}$$

$$\sin\theta = \sqrt{1-\frac{c^4}{u^2v^2}}.$$

$$\tan\theta' = \frac{u_y'}{u_x'} = \sqrt{1-v^2/c^2}\cdot\frac{u\sin\theta}{u\cos\theta+v}$$

$$\Rightarrow \text{transmute} \quad \sqrt{1-v^2/c^2}\cdot\frac{u\cdot\sqrt{1-\frac{c^4}{u^2v^2}}}{-\frac{c^2}{v}+v}$$

$$= \frac{\sqrt{1-v^2/c^2}\ \sqrt{u^2-c^4/v^2}}{v\left(1-c^2/v^2\right)}$$

$$= \frac{\sqrt{1-v^2/c^2}\ \sqrt{u^2-c^4/v^2}}{\frac{1}{v}\left(v^2-c^2\right)}$$

$$= -\sqrt{1-v^2/c^2}\cdot\frac{uv\sqrt{1-c^4/u^2v^2}}{c^2\left(1-v^2/c^2\right)}$$

$$= -\sqrt{1-v^2/c^2}\ \frac{uv}{c^2}\ \frac{\sqrt{1-\frac{c^4}{u^2v^2}}}{1-v^2/c^2}$$

$$= -\frac{uv}{c^2}\ \frac{\sqrt{1-\frac{c^4}{u^2v^2}}}{\sqrt{1-v^2/c^2}} \xrightarrow{v\to c} -\frac{u}{c}\ \frac{\sqrt{1-c^2/u^2}}{\sqrt{1-v^2/c^2}}$$

$$\to -\frac{u}{c}\ \frac{\sqrt{1-c^2/u^2}}{\sqrt{2}\sqrt{1-v/c}} \to -\infty \text{ as } v\to c$$

so asymptotes → ⊥ x-axis